STRATEGIES FOR SEQUENCES AND SERIES

If a sequence \( \{ a_n \} \) has a limit \( L \), then the sequence is said to converge to \( L \).

If a series converges, then \( a_n \) approaches 0 as \( n \) increases (goes to \( \infty \)). \( a_n \) must approach 0 as \( n \) increases to \( \infty \) for a series to converge. The converse is not necessarily true.

The Alternating Series Test is easy to use since it requires that (1) only consecutive terms must be compared \( |a_{n+1}| \leq |a_n| \) and (2) the limit of the \( n \)th term is zero. Both conditions must hold.

For the Comparison Test to work, it is only necessary for the terms to be eventually less than the convergent terms (or greater than the divergent terms) for all terms after a certain value for \( n \). A finite number of terms which are not less than the corresponding terms in the convergent series will not affect the convergence.

The Comparison Test will only be conclusive if you show that a series is:
1. less than a convergent or
2. greater than a divergent

Proving that a series is greater than a convergent series is inconclusive.

If \( p = 1 \), the \( p \)-series is the harmonic series. Convergence for other \( p \)-series can be tested by the Integral Test.

Certain tests are more helpful for certain forms. Terms that can be integrated easily suggest the Integral Test. Factorial notation generally lends itself to the Ratio Test. A sequence that does not converge to zero may suggest the \( n \)th term test.

Absolute convergence implies conditional convergence. If \( \sum |a_n| \) converges, then \( \sum a_n \) must also.

If \( \sum a_n \) converges but \( \sum |a_n| \) does not, then the series is conditionally convergent.

The alternating harmonic series converges by the Alternating Series Test; yet the harmonic series diverges. This is an example of conditional convergence.

Special types of series such as geometric, \( p \)-series, telescoping, or alternating are useful for comparison.